Video feedback control of inverted pendulum

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Abstract. The aim of this work is to examine the possibility of fast on-the-fly image processing and regulation of unstable system at once, using simple embedded smart camera with digital signal processor ADSP-BF533 and CMOS image sensor MT9V032. The convolution based algorithm was designed to determine position of inverted pendulum as well as the method of processing measured signals. For positioning of inverted pendulum, a discrete LQ regulator was implemented into the digital signal processor. One of the parts of this paper deals with mechanical parameters of inverted pendulum and optical aberrations of used lens. Results of discussion are used in real construction of system. Designed system is able to work independently and stabilize inverted pendulum using only embedded smart camera without any other sensors nor other processing in PC.

Keywords
Video feedback, image processing, inverted pendulum, digital signal processing

1. Introduction

Almost every day we come across with systems which need to be measured or controlled by camera. In the automation field there are plenty of applications from simple object counting through quality control to control systems where one or more cameras provide data to systems with many actuators.

The main idea of this work is to determine feasibility of regulation of unstable system using embedded camera with internal image processing.

The first step is to derive the motion equations of considered system using Lagrange’s equations.

Then it is necessary to discuss ability of used embedded camera to handle this experiment and also discuss mechanical parameters of inverted pendulum.

Next step is to design and implement algorithms of image processing with respect to speed and low memory consumption.

The last task is to determine parameters of the system, design and implement an appropriate regulator and prompt tests.

2. State space of the system

Lagrange’s equations were used to determine the motion equations of considered system which is shown in the figure 1.

Derived motion equations can be seen below. They are dealing with friction at joint of pendulum $\delta k$ and friction on wheels of cart $\delta v$.

\[
\dot{\varphi} \cos \varphi + g \sin \varphi + \frac{4}{3} l \ddot{\varphi} = -\delta_k \frac{\dot{\varphi}}{m_1 l}
\]

\[
x: (m_1 + m_2) \ddot{x} + m_1 l \ddot{\varphi} \cos \varphi - m_1 l \dot{\varphi}^2 \sin \varphi = F - \delta_v \dot{x}
\]

For driving the cart, a DC brushed motor was used so it is necessary to imply it into the motion equations. By doing this and linearize the system we will get state space of the whole system, where state variables and matrices are

\[
x_1 = x, x_2 = \dot{x}, x_3 = \varphi, x_4 = \dot{\varphi},
\]

\[
A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix},
B = \begin{bmatrix} 0 \\ b_{21} \\ 0 \\ b_{42} \end{bmatrix},
C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},
D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]
where
\[ a_{22} = -\frac{R_l L_j}{l_j} \quad a_{21} = -\frac{R k_t + k_e k_m}{l_j} \]
\[ a_{42} = \frac{3}{4l} \quad a_{43} = \frac{3g}{4l} \quad a_{44} = -\frac{3d e}{4m l} \]
\[ b_{21} = \frac{r k_e}{l_j} \quad b_{24} = \frac{r k_e}{l} \].

R denotes resistance, L is inductance of winding of motor, J is inertia of rotor, r is radius of pulley driving the cart and \( k_t, k_e \) and \( k_m \) are the friction, electrical and inertia constants of motor.

3. Used hardware

To grab the picture, process the image and control the inverted pendulum, it is necessary to use high speed camera and process data in relatively powerful processing unit. In laboratory of videometric, there was developed platform of embedded smart cameras containing all the essentials. The main processing unit is a digital signal processor ADSP-BF533 Blackfin which is a good choice for the image processing. Into the main DSP it is directly connected the CMOS sensor MT9V032 with WVGA (752x480px) resolution. Thanks to the close interconnection between sensor and the processing unit it is possible to achieve a high speed of a data transfer and therefore short delays from capturing data to the action on the actuator.

Fig. 2. Used hardware

Regarding the used camera, it is necessary to design appropriate mechanics of pendulum. Several criteria were considered such as travel distance and length of the pendulum with the respect to the resolution of camera, DC motor momentum with respect to the mass of the pendulum etc.

4. Image processing algorithm

One of the advantages of the used system is a possibility to reduce the amount of data transferred per each frame. All CMOS sensors can work in Region Of Interest (ROI) mode where only part of the image is transferred and also skipping of rows or columns can be used. These features were used and transfer of all data needed for the image processing was reduced to couple of rows of the image. Connecting this with fast on-the-fly image processing method which processes data from one row when another row is transferred enables entire image processing algorithm to be very fast.

The main job of the image processing is to detect the pendulum and to determine its position and angle. Used DSP is optimized for operations like summing and multiplying so the convolution based algorithm was designed. From the knowledge that the black pendulum has appropriate width and it is moving against white background, the convolution mask was designed as
\[ g = (-2,0,1,\ldots,n\ldots,1,\ldots,n\ldots,1,0-2), \]

where n is appropriate number of zeros coming from width of pendulum.

In the figure 3. there is captured the scene with the pendulum as it is seen by camera. Below in the figure 4. is the output from the detection algorithm.

Fig. 3. Real scene with pendulum and other objects

Fig. 4. Output from the algorithm

The DSP needs 612 \( \mu s \) to process two rows which are necessary to determine the position and the angle of the pendulum. To this time we need to add the exposition time of CMOS sensor which is usually much longer then processing time. Nevertheless sampling frequency of pendulum can be up to 500 \( fps \). Appropriate closed loop bandwidth of system is considered in next part dealing with design of regulator.

Since the camera rely on an artificial light it is necessary to think about blinking of lightings. Every lighting is blinking with frequency 100Hz so every image has really different brightness. Algorithm described above is robust enough to work under wide range of brightness.

5. Design of regulator

The design of the regulator cannot start without exact knowledge of the system parameters. So all the parameters were determined, appropriate nonlinear model of the system was made and then the behavior of the model was compared with the real motion of the system.

The Inverted pendulum is the system where two parameters need to be regulated. The first is an angle of the pendulum and the second is a position of the cart. To avoid double feedback control using two regulators, the full state
feedback controller with integral action was used. This was realized by LQ regulator which is defined as follows:

\[
J = V(x(0), u_0^{N-1}, 0) = \frac{1}{2} x^T(N) Q x(N) + \frac{1}{2} \sum_{k=0}^{N-1} [x^T(k) u^T(k)] [Q \quad S^T \quad S \quad R] [x(k) \quad u(k)].
\]

From another derivation we can get the Riccati’s equations which can give us the full state feedback. To design the LQ regulator we need to think about importance of each state and consider it during design. The most important variable is an angle of the pendulum and then the position of the cart. Simulated behavior of the system reacting on impulse disturbance can be seen in the figure 5.

Closed loop bandwidth of system was calculated as 1426 rad/s which corresponds to 227 Hz and therefore desired sampling frequency about 500 fps is sufficient to control the system.

6. Implementation of LQ regulator

The LQ regulator is one of the full state feedback controllers so it is relatively easy to implement it. Action \( U \) for the actuator can be calculated as follows:

\[
U = -(x K_1 + \dot{x} K_2 + \varphi K_3 + \dot{\varphi} K_4 + x_i K_5)
\]

where \( K_i \) are the coefficients of the regulator.

A little bit more complex it is to determine the states of the system. The angle and the position of pendulum are calculated directly from the image processing but a speed of cart and an angular speed of pendulum needs to be calculated as first derivative. Unfortunately measured states are full of noises and simple replace of derivative by differentiation cannot be done. More over all the known filtering methods employ traffic delay which has a negative impact on robustness of the regulator. To get the first derivative and reduce the noise, the convolution with mask approximating first derivative of gauss function was used. Then the speed of cart and angular velocity were calculated.

\[
\dot{x}[k] = \frac{-x[k - 3] - x[k - 2] + x[k] + x(k)}{4T_s}
\]

where \( T_s \) is sampling frequency.

After the implementation of the regulator into DSP several tests of stability of the system were made. Motion data of one of the tests with external disturbance which causes an angle of the pendulum to be about 30° can be seen in the figure 6.

7. Conclusion

At this work was proved ability of simple embedded smart camera to control unstable system. The considered system of inverted pendulum was modeled by motion equations and then built with respect to parameters of smart camera. In the next step was developed algorithms for fast on-the-fly image processing on digital signal processor calculating states of the system. This algorithm is based on convolution and can deal with blinking of lightings. Then was determined the parameters of the system and designed LQ regulator which is able to stabilize the inverted pendulum. Whole system was tested under different light conditions and other disturbances.
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References


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Jiří HLADÍK was born in Turnov, Czech Republic in 1990. He received his master’s degree in 2015 from Cybernetics and robotics at Czech Technical University. At present he continues in his PhD studies at the same university focusing on precise measurement of position.