Abstract. Human-computer communication in a natural language would make modern information technologies available to the general public. To achieve this goal, it is necessary to process the natural language semantics. Formalizing and interpreting the sense of the natural language sentences through some logical formalisms is called logical analysis of the natural language. In such logic analysis, intensional logic systems are usually used. This article discusses historical developments in logical analysis of natural language, and the reasons for using intensional logic instead of traditional extensional logic.

Keywords

extension, Frege’s semantic triangle, intension, logical analysis of the natural language sentences, possible worlds

1. Introduction

The idea, spread by the Stoics more than two thousand years ago, which talked about the distinction of a particular thing, its designation and its way of designation, was once again “revived” in modern theoretical semantics by Gottlob Frege. Let’s have a model example of Eleonora of Aquitaine’s life. The queen Eleonora of Aquitaine can be identified in several ways:

(1) Eleanor of Aquitaine was the wife of king Henry II of England.
(2) Eleanor of Aquitaine was the mother of king Richard the Lionheart of England.

Both characteristics “being the wife of king Henry II of England”, “being the mother of king Richard the Lionheart of England” clearly identify Eleanor of Aquitaine, because Henry II had only one wife and Richard the Lionheart had only one mother. Frege, in his classical writings called Über Sinn und Bedeutung [1], called the method of identifying an object as a sense of the expression, and the identified object

as a reference of the expression\(^1\). Fig. 1 shows a well-known Frege’s semantic triangle.

\[\text{Expression} \xrightarrow{\text{designate}} \text{Reference} \xrightarrow{\text{determine}} \text{Sense} \xrightarrow{\text{express}} \text{Expression}\]

Fig. 1. Frege’s semantic triangle - definition

We can define the terms of Frege’s semantic triangle in the following way:

- expression - linguistic form, term, symbol,
- reference - real world object,
- sense - mode of presentation, thought, concept.

The characteristics “being the wife of king Henry II of England” and “being the mother of king Richard the Lionheart of England” are used to designate the same reference, but they express a different sense [2]. Fig. 1 captures the location of the sentences (1), (2) into the Frege’s semantic triangle.

\[\text{“the wife of King Henry II of England.”} \xrightarrow{\text{designate}} \text{Eleanor of Aquitaine} \xrightarrow{\text{determine}} \text{queen as wife}\]

\[\text{“the mother of Richard the Lionheart”} \xrightarrow{\text{designate}} \text{Eleanor of Aquitaine} \xrightarrow{\text{determine}} \text{queen as mother}\]

Fig. 2. Frege’s semantic triangle - application

2. Extensional principles

Frege’s distinction between reference and sense has led to the conclusion that the semantics of the natural language

\(^1\)Sometimes the term “meaning” is used instead of “reference”.

is two-dimensional [3]. It means that each expression is designate by its reference and expresses some sense. Propositional logic (PL) and first-order predicate logic (FOPL) are logical systems based on this extensional principle. It means that the expression always refers to only one reference with the corresponding truth value.

The original Frege’s semantic triangle represented by Fig. 1 can be extended to the form shown in Fig. 2.

![Extended form of Frege’s semantic triangle](image)

**2.1. Extensional principle of compositionality**

Formal representation of sense of the natural language expression should not only include its components, but also information about its logical structure. Consider the expressions:

(3) “small car in a large garage”,

(4) “big car in a small garage”.

The sense of expressions (3), (4) is not the same. For PL and FOPL logic systems, it is necessary to formulate Frege’s extensional principle of compositionality: The meaning of a compound expression is determined by the meaning of its meaningful sub-expressions and the way of their logical connection.

**2.2. Extensional principle of substitution**

Consider the sentence:

(5) The article needs a language correction.

If we replace the expression „language correction“ with expression „text editing“, we get the sentence

(6) The article needs a text editing.

which has the same sense as (5). Therefore, Leibniz’s law can be generalized to the extensional principle of substitution: The meaning of the compound expression is preserved after its sub-expression is replaced with other sub-expression of equivalent meaning.

**3. Intensional principle**

Let’s go back to Fig. 1, according to Eleanor of Aquitaine holds two different functions - she is the wife of Henry II and she is also the mother of Richard the Lionheart. These functions are called individual roles [4] and allow us to identify a reference. Before Henry II of England, Eleanor of Aquitaine was a wife of Louis VII of France, we can say:

(7) Eleanor of Aquitaine was the wife of king Louis VII of France.

However, individual roles ”being the wife of king Louis VII of France” from sentence (7) does not necessarily refer to the reference of Eleanor of Aquitaine, because Louis VII was married also with Constance of Castile and Adele de Champagne. It means that the individual who occupies that individual role has changed over time. Similar temporal dependence can be identified in an individual role ”being the husband of Eleanor of Aquitaine”, which designates the reference of Louis VII and reference of Henry II. If it is possible for multiple references to refer to the same expression, we will call them a denotate of expression. Applying the terminology of Ludwig Wittgenstein [5], the denotate of the individual role depends on changing states of affairs. This explains why a person understands the expression even though he does not know his particular denotate (it is not necessary to know who is Peter Sagan to understand to the expression ”being the world champion in road cycling”).

**3.1. Possible worlds**

Eleanor of Aquitaine has always been Eleanor of Aquitaine - the individual can not turn into another individual. However, she could have other husbands besides the king of England and king of France, and her son Richard the Lionheart could have a completely different mother. The fact that these events did not happen in the real world does not mean that they are not logically possible. The maximum set
of logically possible facts (empirical characteristics of objects) is considered to be one logically possible state of the world. The individual can take different features in different worlds that change his individual role. In the 18th century such states of the world were called possible worlds by Gottfried Wilhelm Leibniz [6]. Leibniz wondered why God allows so much misery. He has come to the view that God gives people the ability to live in the best of the worlds that are possible. In the 20th century, the term “possible world” began to be used by Rudolf Carnap [7] in the field of theoretical semantics and modal logic.2

The possible world is the time sequence of a completely consistent set of valid facts. The property of completeness in this definition means that every world contains all the possible facts that are either true or false. Consistency speaks of the integrity of these facts - the world can not contain two opposing statements that are both true.

If we go back to historical facts, Eleanor of Aquitaine had only two husbands in the real world, and Richard the Lionheart was her son. The actual world is the time sequence of a completely consistent set of real facts. It is not possible to find out and prove which one of the possible worlds is the actual world. Identifying the actual world would presuppose logical omniscience - the need to know all the real facts of the actual world.

Possible worlds are therefore abstract entities that describe not only what things really are but also what they might be. They are alternatives to the real world. The concept of possible worlds has removed some disadvantages of traditional logic. PL and FOPL are not designed to describe the meaning of natural language expressions because they can not capture possible variants of reality. They can describe only one static state of the dynamically changing world.

3.2. Intensional version of Frege’s thoughts

Natural language contains expressions whose sense is not same in every possible world and time point, and expressions whose sense is the same in all circumstances. Expressions independent from state of world are:

- names of individuals (Henry II, Peter Sagan...),
- mathematical expressions ($2 + 2 = 4, x \in \mathbb{R}$...),
- logical symbols ($\forall, \exists, \&, \lor$...).

The fact that mathematical expressions and logical symbols are independent of a particular possible world is obvious. To explain why the individuals have the same property, consider the following sentence:

(8) If Eleanor of Aquitaine was not the heir of Aquitaine, she would not become the wife of king Louis VII of France.

Sentence (8) speaks of a possible world in which the Eleanor of Aquitaine is present, and if this individual does not have the property “being the heir of Aquitaine” it would not have the property “being the wife of Louis VII”. However, such an analysis can only be made if the world contains an individual of Eleonora of Aquitaine with its specific characteristics, not just any individual with the same name. This means that the same individual exists in all possible worlds and is independent of them3.

Rudolf Carnap proposed to use the terms extension and intensions, instead of Frege’s reference and sense. Extension is what two equivalent expressions have in common. For equivalence of expression from Fig. 2, we can say:

- individual concepts $a, b$ are equivalent if $a$ is the same individual as $b$,
- predicates (properties and relationships) are equivalent if the sets of objects to which they refer contain the same elements,
- sentences are equivalent if they have the same truth value.

Intension is what two logically equivalent expressions have in common. The sentences $A, B$ are logically equivalent when the sentence $A$ is true under the same circumstances in which the sentence $B$ is true. This means that the sentences are logically equivalent when they have the same true value in the same state of affairs or possible worlds at time points.

The truth value of expressions in an extensional context does not depend on the possible world and time point. All other expressions of natural language are intensions. Knowing an extension of an expression means knowing what is the reference of an expression in the actual world [9]. Knowing what is an intension of an expression is to determine its denotate - its extension in all possible worlds. Therefore, intension of an expression can be described as an extension relative to possible worlds. The intensional individual role (“being the wife of Louis VII”) can be understood as a function that assigns to every possible world and every time point the chronology of the denotates (Eleanor of Aquitaine, Constance of Castile, Adele de Champagne, ...).

2Despite the fact that the theory of possible worlds was generally accepted, there were also several critics. The most famous of them is Willard Van Orman Quine. Because of the paradoxes arising from the semantic interpretation of modal logic with quantifiers, he rejected this theory.

3This explanation of the individual nature refers to the intensional logic used in logical analysis of the natural language. Opposite to this approach, David Lewis [8] and his followers believe that the individual exists exclusively in one world from a set of possible worlds. In other possible worlds there are counterparts of such individual - individuals that are so similar that we can identify them with the original individual in semantic analysis. The theory of counterparts is based on these foundations.
It is obvious that Frege’s perception of the natural language semantics was extensional. We can define intension of an expression as follows:

*Intension of expression is a function* \( s(w, t) = d \), *which assign every state* \( s \) *specified by the possible world* \( w \) *and the time point* \( t \), *chronicle of denotate* \( d \).

Based on this definition, proposition of expression is a function from a set of the possible worlds in time points to a set of truth values. Using the notion of a possible world, we can change Frege’s extensional semantic triangle from Fig. 1 into its intensional form - Fig. 3.2.

**Fig. 4. Intensional semantic triangle**

The notions from intensional semantic triangle 3.2 are characterized in the following way:

- denotate of an expression - set of references,
- sense of expression relativized to a set of the possible worlds in time points - sense of expression is expressed through a function with two arguments - sets of possible worlds and time moments.

We can reformulates the extensional principle of compositionality 2.1 into intensional way: *The sense of a compound expression relativized to a set of the possible worlds in time points is determined by the sense of its meaningful sub-expressions relative to this set and the way of their logical connection.*

Similarly, we can reformulate the extensional principle of substitution 2.2: *The sense of a compound expression relativized to a set of the possible worlds in time points is preserved after its sub-expression is replaced with other subexpression from that set of an equivalent sense.*

### 4. Conclusion

In this article, we have tried to explain why it is not appropriate to use extensional logical systems for the interpretation of the natural language sense. Understanding the historical transition from the extensional character of the natural language to the intensional is essential for further investigation of the systems used for logical analysis of the natural language. In the next steps of our research, we will try to find such an intensional logic that completely overcame the imperfections of the extensional logic systems - contextualism, non-universality or the impossibility of examining a wide spectrum of semantic phenomena.

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### References


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