Projection GDP using Input-output model

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Abstract. *Input-output analysis is powerful for modeling global economies or industries. The Leontiev Input – Output economic models show the relationships among different sectors of industry in an economy. These Input - Output models, whether closed or open Leontiev models, represent miscellaneous types of economies. It is possible to tell or forecast not only all sector development given economy, it is also possible forecast development of Gross domestic product by solving the Leontiev open models. Input - Output analysis is a valuable tool for GDP projections into the future.*

Keywords

Economy, Leontiev, Input – Output analysis, consumption, demand, industry sectors, GDP.

1. About Vasily Leontiev

American economist Vasily Leontiev (as well as Wassily Leontief) was born in 1906 in Munich and he grew up in St. Petersburg. In 1921 he entered University of Leningrad. He studied first philosophy and sociology, and then economic sciences. After graduating in 1925 he continued his education at the University of Berlin. In 1927 while still a student, he began his professional career as a research assistant at Kiel University. At the age of 22 he received a doctorate in economics. After emigrating in 1931 in the United States, he joined the National Bureau of Economic Research and began his long work in the United States at Harvard University as a teacher of Economics. In 1946 he became a full professor.

To solve the problem of economic growth and development, Leontiev beside evolved a dynamic version of the first static analysis model Costs - Issue (as well as Input - Output analysis) and then he added the indicators of capital requirements to the list of so-called final demand, or final sales. A success the application of models of economic analysis Costs - Issue in no small measure due to his outstanding abilities. Since Leontiev research economists over the whole world have different possibilities of application in of this model, such example, as the theory of international trade, theory of monopoly, econometrics.

Leontiev was awarded the Nobel Memorial Award in Economics in 1973 for the development of the method Costs - Issue and for its application to important economic problems. As one of the first economists concern at the impact of economic activity on environmental quality, Leontiev quoted in his Nobel lecture, a simple model of Costs - Issue, referring to the global environment, in which pollution is clearly figured as a separate sector. In less developed countries, - he concluded - the introduction of softening of the strict standards against pollution of the environment will increase employment, although it will require some of the victims in the sphere of consumption [3].

2. Input - Output method

Leontiev Input - Output model normally have a large number sectors of industry and it will be quite complicated. For a simplification, the following assumptions are adopted:

- each industry produces only one homogeneous commodity;
- each industry uses a fixed input ratio for the production of its output;
- production in every industry is subject to constant return to scale (constant returns to scale means k-fold change in every input will result in an exactly k-fold change in output) [1].

The Leontiev model represents the economy as a system of linear equations and its has a tree components: matrix of internal sector demand (**A**), matrix of final/external demand (**d**) and matrix of total production (**P**). Elements of these matrices could show as units of product or price value. To find production vector (**P**) in terms of demand vector (**d**), we solve system of linear equations. Such equations are naturally represented using the formalism of matrixes and vectors and we solve system of linear equations with help of matrix algebra and Gaussian-elimination method.

2.1 The Open Leontiev Model

There is a closed Leontiev Model where no production leaves or enters the economy, which means there is no external demand, in more details [1]. In real world, it does not happen very often. Usually common economy has external or final demand. Hence, for prediction GDP analysis we use the open Leontiev Model.

Final demand

In Open Leontiev Model each industry has a demand for products from other industries (internal demand of each industry). Also, there are external demands from outside. Model finds a production level for the industries that will supply both internal and external demands.

We assume there are *i* interdependent sectors of industry: $S_1, S_2, \ldots, S_i$.:

- m_{ij}: the units of product produced by industry S_i to produce one unit of industry S_i ;
- p_i: the production level of industry S_i ;
- $m_{ij}p_j$: the number of units produced by industry S_i and consumed by industry S_j ;
- d_i: external demand from the outside industry S_i .

Then, for total number of units produced by industry S_i we can get linear equations:

$$p_1 = m_{11}p_1 + m_{12}p_2 + \ldots + m_{1j}p_j + d_1$$

$$p_2 = m_{21}p_1 + m_{22}p_2 + \ldots + m_{2j}p_j + d_2$$

. .

$$p_i = m_{i1}p_1 + m_{i2}p_2 + \ldots + m_{ij}p_i + d_i \qquad (1).$$

In array expression we can have matrix **A**, vectors **P**, and **d** as **P = AP + d** (2),

$$A = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1j} \\ m_{21} & m_{22} & \cdots & m_{2j} \\ \cdots & \cdots & \cdots & \cdots \\ m_{i1} & m_{i2} & \cdots & m_{ij} \end{bmatrix}, \quad P = \begin{bmatrix} p_1 \\ p_2 \\ \cdots \\ p_i \end{bmatrix}, \quad d = \begin{bmatrix} d_1 \\ d_2 \\ \cdots \\ d_i \end{bmatrix}$$

Matrix **A** is called Input - Output matrix or consumption matrix and it shows the quantity of inputs needed to produce one unit of product. The rows of the matrix represent the producing sector of the economy. The columns of the matrix represent the consuming sector of the economy. The entry m_{ij} in consumption matrix represents what percent of the total production value of sector *j* is spent on products from sector *i*. Vector **d** is the demand vector, which represents demand from the non-producing sector of the economy (external demand). Vector **P** represents the total amount of the product produced in industry.

2.2 Projection GDP and Input - Output model

Gross domestic product (GDP) is used commonly to measure the economic performance of a whole country or economy, but can also measure the relative contribution of any industry sector S_i. GDP can be allocates in three ways and all of them give same value. They are the production (know also output or value added) method, the income method, and the expenditure method [7].

In the expenditure approach estimating of GDP we separate four components: the sum of consumption (C), investment (I), government expenditure (G) and net exports (Ex – Im).

$$GDP = C + I + G + (Ex - Im) \qquad (3)$$

For GDP projection we have to divide from (2) final demand **d** to the external domestic demand $\mathbf{d}^D$ and amount of product dedicated to export outside of economy $\mathbf{d}^E$. In same case we have to total production **P** divide to the total production level of domestic consumption $\mathbf{P}^D$ and total amount of imported equivalent product for internal demand $\mathbf{P}^I$. input output model. And final we distribute internal demand **A** resp. $\mathbf{A}^D$ from domestic industry sectors S_j to internal demand of equivalent imported product $\mathbf{A}^I$. This adjustment of basic input - output model come to better utilization of input – output model and lead to a new statement in array expression:

$$\mathbf{P}^D + \mathbf{P}^I = \mathbf{A}^D\mathbf{P}^D + \mathbf{A}^I\mathbf{P}^D + \mathbf{d}^D + \mathbf{d}^E \qquad (4)$$

- $\mathbf{P}^D$ the vector of total production level for domestic consumption produced by each industry S_i,
- $\mathbf{P}^I$ the vector of imported equivalent product for internal demand each industry S_i,
- $\mathbf{A}^D$ the internal consumption matrix and it shows the quantity of domestic inputs needed to produce one unit of domestic product;
- $\mathbf{A}^I$ the internal consumption matrix and it shows the quantity of imported inputs needed to produce one unit of domestic product;
- $\mathbf{d}^D$ the external demand from the outside industry S_i consumed in domestic economy,
- $\mathbf{d}^E$ the final demand from the outside industry S_i dedicated to export outside of economy.

Then using (3) and (4) we can identify total domestic demand $\mathbf{d}^D$ and net export.

$$\Sigma d_i^D = C + I + G \qquad (5)$$

$$(\Sigma d_i^E - \Sigma p_i^I) = (Ex - Im) \qquad (6)$$

In the model (4) is not reflect relation between domestic production p_i^D and imported product p_i^I. This relation can be described as constant k_i for each industry sector S_i:

$$k_i = p_i^I / p_i^D \qquad (7)$$

The input – output model (3) is simplified to:

$$(\mathbf{I} + \mathbf{k})\mathbf{P}^D = \mathbf{A}_D\mathbf{P}^D + \mathbf{A}_I\mathbf{P}^D + \mathbf{d}^D + \mathbf{d}^E \qquad (8)$$

To solve this system of linear equations, we can find solution and answers from (8) [2]:

$$\mathbf{d}^D = (\mathbf{I} + \mathbf{k} - \mathbf{A}_D - \mathbf{A}_I)\mathbf{P}^D - \mathbf{d}^E \qquad (9)$$

$$\mathbf{d}^E = (\mathbf{I} + \mathbf{k} - \mathbf{A}_D - \mathbf{A}_I)\mathbf{P}^D - \mathbf{d}^D \qquad (10)$$

$$\mathbf{P}^D = (\mathbf{I} + \mathbf{k} - \mathbf{A}_D - \mathbf{A}_I)^{-1}(\mathbf{d}^D + \mathbf{d}^E). \qquad (11)$$

2.3 Application of input – output method

In this paper we use same data example [6]. i.e. we selected two sectors, sector of electricity industry and coal industry in year 2009. We suppose all imported coal is consumed for produce electricity, no imported coal is use for produce coal. Only 1 % imported electricity is consume for produce coal as common consumption, rest of imported electricity is consumed on external domestic market.

In our paper we want to show the interdependence between these two sectors and highlight the changes in GDP contribution from selected sectors, that the coming variation in demand. Data about sectors was taken from [4] and [5].

All units in TJ		Internal demand from production sector		Internal demand imported equiv. product	
Production sector of industry		S_1^D	S_2^D	S_1^I	S_2^I
S_1	Electricity industry	22 536	6 453	0	309
S_2	Coal industry	105 589	52 069	58 831	0
		External demand from non -production sector		Total domestic production (P_i^D)	Imported equivalent production (P_i^I)
Production sector of industry		Domestic d_i^D	Export d_i^E		
S_1	Electricity industry	217 684	80 028	296 100	30 910
S_2	Coal industry	512 033	179 979	849 669	58 831

Tab. 1. Production - demand of selected sectors

From Tab.1. we can calculate by the (5) and (6) contributory value to GDP

ΔGDP = 217 375 + 512 033 + (80 028+179 979 – 30 910 – 58 831)

ΔGDP = 899 674 in TJ

This value is in TJ, which means we eliminate monetary problem base-point pricing of all products from industry sectors. Using TJ unit lucidity of contributory value to GDP is not affect.

For finding solution of this model we have to calculate Leontiev matrix by (4)

$$(\mathrm{I} - \mathrm{A}^{\mathrm{D}} - \mathrm{A}^{\mathrm{I}}) = \begin{bmatrix} 0.92389 & -0.00796 \\ -0.55529 & 0.93872 \end{bmatrix}$$

$$(\mathrm{I} - \mathrm{A}^{\mathrm{D}} - \mathrm{A}^{\mathrm{I}})^{-1} = \begin{bmatrix} 1.08792 & 0.00922 \\ 0.64355 & 1.07074 \end{bmatrix}$$

Case 1. *What change occurs in model if we expect electricity import dropped by 5 %? Domestic demand is the very same.*

The answer to this question we can find by open Leontiev's model Input – Output analysis. We found a new total domestic demand vector $\mathbf{P}^{\mathrm{D}}$* according to (4) and new consumption matrices $\mathbf{A}^{\mathrm{D}}$* and $\mathbf{A}^{\mathrm{I}}$* in natural units TJ

$$P^{D*} = (I - A^D - A^I)^{-1}(d^D + d^E - P^I) = \begin{bmatrix} 297\,781 \\ 850\,664 \end{bmatrix}$$

$$A^{D*} = \begin{bmatrix} 22\,664 & 6\,460 \\ 106\,189 & 52\,130 \end{bmatrix}, \quad A^{I*} = \begin{bmatrix} 0 & 309 \\ 59\,165 & 0 \end{bmatrix}$$

Now we calculate a new contributory to GDP. ΔGDP = 901 528 in TJ, we see this contribution increased by 0.17 %. This increasing is only by dropping import, but industry sectors are more motivated for increasing own domestic production (**A**). This we can call future effect.

In our model, when we reflect relation between domestic production $\mathbf{P}^{\mathrm{D}}$ and imported product $\mathbf{P}^{\mathrm{I}}$, we enumerate vector **k** by (7):

$$k = \begin{bmatrix} 0.1044 \\ 0.0692 \end{bmatrix}.$$

We need to calculate a new matrix $(\mathbf{I+k\text{-}A^D\text{-}A^I})$ and its inverse matrix $(\mathbf{I+k\text{-}A^D\text{-}A^I})^{-1}$

$$(\mathrm{I} + \mathrm{k} - \mathrm{A}^{\mathrm{D}} - \mathrm{A}^{\mathrm{I}}) = \begin{bmatrix} 1.02828 & -0.00796 \\ -0.55529 & 1.00796 \end{bmatrix}$$

$$(\mathrm{I} + \mathrm{k} - \mathrm{A}^{\mathrm{D}} - \mathrm{A}^{\mathrm{I}})^{-1} = \begin{bmatrix} 0.97666 & 0.00771 \\ 0.53804 & 0.99635 \end{bmatrix}$$

Case 2: *What change occurs in model if we expect external domestic consumption of electricity rise 1 %?*

Also the answer to this question we can find by open Leontiev's model Input – Output analysis. We found a new total domestic demand vector $\mathbf{P}^{\mathrm{D*}}$ according to (11) and vector of imported production $\mathbf{P}^{\mathrm{I*}}$:

$$P^{D*} = (I + k - A^D - A^I)^{-1}(d^D + d^E) = \begin{bmatrix} 298\,226 \\ 850\,841 \end{bmatrix}$$

$$P^{I*} = k \times P^{D*} = \begin{bmatrix} 31\,132 \\ 58\,912 \end{bmatrix},$$ and new consumption matrices $\mathbf{A}^{\mathrm{D}}$* and $\mathbf{A}^{\mathrm{I}}$* in natural units TJ:

$$A^{D*} = \begin{bmatrix} 22\,698 & 6\,462 \\ 106\,347 & 52\,141 \end{bmatrix}, \quad A^{I*} = \begin{bmatrix} 0 & 310 \\ 59\,253 & 0 \end{bmatrix} \text{[TJ]}$$

In this case we numerate ΔGDP = 901 857 in TJ. In the second case, we can see that although the external domestic consumption of electricity increased by 1 % contribution of GDP increased only by 0,21 %. That means positive effect of domestic consumes and negative effect of imported equivalent products.

3. Conclusion

In our paper we describe the use of Leontiev's matrix and simple cases demonstrating their potential use for analysis GDP contribution and possible scenarios of future development of the various sectors of industry or economy. Those simplified cases in our paper shows a briefly point of two industry sectors in year 2009. We analyzed the data with help of matrix algebra and Leontiev open model and calculate changes in GDP contribution. The relationship between changes in selected industry sectors and GDP is inconsiderable, but in whole economy, where relationships between sectors are very affinity, is GDP contribution significative.

This paper shows potential of Leontiev input – output models. Whole economy of the state is more complicated, consequently demands and consumptions coefficients are always changing. Leontiev models can be fitted to more exactly reflect this and be used in the government to predict the economic and environmental impacts such GDP.

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Josef ČERNOHOUS, born in Lanškroun in 1976. Successfully graduated from FEE, CTU in Prague in February 2005, specializing Economics and Management of Electrical Engineering, diploma thesis was Wage structure and development in CR. Present time combined form of doctoral study at the Department Economics, Management and Humanities FEE, CTU in Prague.